ORIGINAL ARTICLE

ON THE DYNAMIC BEHAVIOR OF UNIFORM RAYLEIGH BEAM WITH AN ACCELERATING DISTRIBUTED MASS

| Ogunyebi Segun Nathaniel [1*] | and | Adedowole Alimi [2] |

[1.] Ekiti State University | Department of Mathematics | Ado Ekiti, Nigeria |
[2.] Adekunle Ajasin University| Department of Mathematical University | Akungba-Akoko, Nigeria |

|Received | 08 March 2017| |Accepted | 15 April 2017| |Published 25 April 2017 |

ABSTRACT

Background: Structural members (Beam and plate) play significant roles in the construction of highway bridges and railroads; hence he need to assess the dynamic behavior of these elements has attracted various researchers in the field of Mathematical Physics and Engineering especially when they are subjected to moving concentrated and moving distributed masses. **Objectives:** The specific objective of this present work is to obtain analytical solutions to the non-homogeneous fourth order partial differential equation governing simply supported Rayleigh Beam with an accelerating distributed mass on elastic foundation**. Methods:** The technique adopted for the uniform forced vibration of Rayleigh beam with accelerating mass resting on variable elastic foundation at constant velocity is Fourier sine transforms and Laplace transformation for the analytical solutions. **Results:** The dynamic effects of vital parameters such as elastic foundation, rotatory inertia correction factor, axial force, distance and load parameters were obtained. **Conclusions:** Solutions to the uniform forced transverse vibration of Rayleigh beam with an accelerating distributed mass is given and the information about the vital parameters are presented.
Keywords: *Axial force, accelerating mass, Elastic foundation, Response, Rayleigh beam.*

1. INTRODUCTION

Studies on the moving load is a great importance in the field of transport and construction engineering and prominent examples of structural elements that support the moving loads are railways, bridges, runways, guide ways, overhead cranes etc. A lot of researches have been conducted during the last one hundred years by the authors in the field of structural dynamics with knotty problems encountered since moving load induces larger deflections and stresses on the structure on which it moves than does on equivalent static load. In the past, the most general type of moving loads in structural studies is a constant or harmonic force and a moving load problem can be aptly defines by using beam or plate subjected to moving point wise mass.

Several investigations have been carried out on the problem of flexural vibrations of elastic structures to moving loads due to advances in transport and automobile engineering which results in high speed and heaviness of vehicle and other moving bodies. Prominent among these authors are Stokes (1883) who solved a problem on an associated mass travelling over a massless beam as it applicable to Railway Bridge [1]. Lee (1996) studied the dynamic response of a beam acted upon by moving load or moving masses in connection with the design of railway tracks and bridges and machining process [2]. The equation of motion in matrix form has been formulated for the dynamic response of a beam acted upon by a moving mass by using the Langragian approach and the assumed mode method and found that the separation of the mass from the beam may occur for a relatively slow speed and small mass when the beam is clamped at both ends. Fryba (1999) presented some fundamentals in the dynamics of structures under moving loads [3]. Kargarnovin and Younesian (2004) investigated the response of a uniform Timoshenko beam of infinite length placed on a viscoelastic foundation and subjected to an arbitrary distributed harmonic moving load in which the speed and frequency of the moving load are assumed to be constant [4].

Moving load can be approximated to be concentrated or distributed. Concentrated load or lumped mass act at appoint on the structure along a single line in space. This has been a common subject of investigation for authors as it is a simplified formulation for moving load problems. Distributed loads on the other hand practically provides more accurate model of the moving load problems as the loads are actually distributed over a small segment or over the entire length of the structural members they traverse Oni and Ogunyebi [5].
Among the authors that have worked on concentrated load problems is Volterra (1965) [6]. He describes a method for analyzing the response of railroad tracks to a moving concentrated load, and determines the maximum dynamic

deflection and theoretical critical speed (1168mph) which is more than ten times the critical speed determined experimentally by Ignis (1934) [7]. Shahin and Mbakisya (2010) investigated the response of a simply-supported beam on elastic foundation to repeated moving concentrated load by means of the Fourier sine transformation [8].

Consequently, concentrated load problems continue to draw attention of authors as Sadiku and Leipholz (1981), Gbadeyan and Oni (1995), and Stanisic et al., (1969) in the field of applied Mathematics and Transport engineering [9,10, 11]. On the other hand, limited number of researchers on vibration of structures for beam, plates and shells with distributed moving loads has appeared in the literatures [12, 13, 14]. Rieker and Trethewey (1999) investigate the finite element analysis of an elastic beam structure under moving distributed load [15].

Resently, Usman et al (2015) examined the vibration of Timoshenko beam subjected to partially distributed moving load. The method of series solution and numerical method were used to solve the equation that governs the system [16].

However, this present work focuses on the flexural behaviour of thick beam with an accelerating distributed mass on variable elastic foundation with uniform velocity. Numerical example involving a simply supported thick beam is studied and effects of some vital parameters are also presented.

2. MATHEMATICAL FORMULATION AND SIMPLIFICATION OF THE GOVERNING EQUATION

The Rayleigh beam theory (1877) provides a marginal improvement on the Bernoulli-Euler theory by including the effect of rotation of the cross-section. As a result, it partially corrects the over estimation of natural frequencies in the Bernoulli-Euler model. The figure below illustrate the dynamic response of a simply-supported Rayleigh beam on variable elastic foundation to accelerating distributed loads

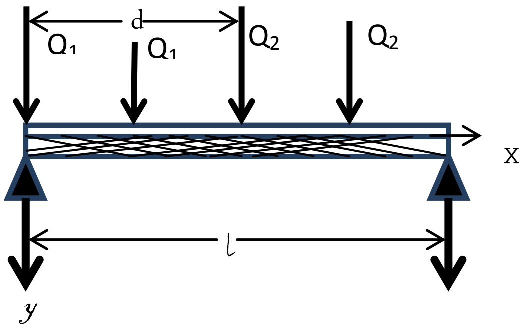

Figure 1: Distributed load on elastic foundation.

Given that the time $t=\frac{x_0}{c}$ measured from the instant that the first load Q_1 is x_0 distance from the left support, the fourth order partial differential equation that govern the transverse motion of simply-supported Rayleigh beam may be written in the form

$$EJ\frac{\partial^4}{\partial x^4}W_z\left(x,\frac{x_0}{c}\right)-N\frac{\partial^2}{\partial x^2}W_z\left(x,\frac{x_0}{c}\right)+\mu_z\frac{\partial^2}{\partial t^2}W_z\left(x,\frac{x_0}{c}\right)$$

$$-\mu_z R^0\frac{\partial^4}{\partial x^2\partial t^2}W_z\left(x,\frac{x_0}{c}\right)+S(x)W_z\left(x,\frac{x_0}{c}\right)=Q_1H(x-ct)+HQ_2[x-(ct+x_0)] \qquad (1)$$

Where

EJ is the flexural rigidity, E the Young modulus, J the moment of inertia, W_z is the displacement, x is spatial coordinate, c is the speed of the moving distributed load, x_o is the distance between repeated moving distributed loads, the axial force is represented by N, μ_z is the mass of the structure, R^0 is rotatory inertia correction factor, $S(x)$ is the variable elastic foundation, time is given by t, Q_i is the moving distributed load and $i=1,2\cdots$. $H(x-ct)$ is the Heaviside function defined as

$$H(x-ct)=\begin{cases}0, & for\ x\le ct\\ 1, & for\ x\ge ct\end{cases} \qquad (1a)$$

The Rayleigh beam is assumed to be continuously supported by variable elastic foundation as defined Ogunyebi (2014)

$$S(x)=S_0(4x-3x^2+x^3) \quad (2)$$

Where S_0 is the constant elastic foundation.

The beam considered in this present work has boundary condition and the initial condition given as

$$W_z(x,t)=0 \quad (3)$$

$$\frac{\partial W_z(x,t)}{\partial x}=0 \quad (4)$$

at $x=0$ and $x=L$.

3. SOLUTION TECHNIQUES

In this section, in order to compute the response of thick beam due to moving distributed load, the non-homogeneous fourth order partial differential equation governing the system is solved by Fourier sine transformation method defined by

$$V_z(n,t)=\int_0^L W_z(x,t)\sin\frac{n\pi x}{L}dx \quad (5)$$

And

$$W_z(x,t)=\frac{2}{L}\sum_{n=1}^{\infty}V_z(n,t)\sin\frac{n\pi x}{L} \quad (6)$$

where $V_z(n,t)$ is the transform of the original equation $W_z(x,t)$.

Substituting equation (5) into equation (1), one obtains

$$\frac{EJ}{\mu_z}\left(\frac{n^4\pi^4}{L^4}\right)V_z(n,t)sin\frac{n\pi x}{L}+\frac{N}{\mu_z}\left(\frac{n^2\pi^2}{L^2}\right)V_z(n,t)sin\frac{n\pi x}{L}+\ddot{V}_z(n,t)sin\frac{n\pi x}{L}+R^0\left(\frac{n^2\pi^2}{L^2}\right)\dot{V}_z(n,t)\cos\frac{n\pi x}{L}$$
$$+\frac{S_0}{\mu_z}\left(4x-3x^2+x^3\right)V_z(n,t)sin\frac{n\pi x}{L}=\frac{1}{\mu_z}\left[Q_1 sin\frac{n\pi ct}{L}+Q_2\sin n\pi\frac{(ct+d)}{L}\right] \quad (7a)$$

By orthogonally condition, equation (7) becomes

$$\frac{EJ}{\mu_z}\left(\frac{n^4\pi^4}{L^4}\right)V_z(n,t)A_a+\frac{N}{\mu_z}\left(\frac{n^2\pi^2}{L^2}\right)V_z(n,t)A_b+\ddot{V}_z(n,t)A_c+R^0\left(\frac{n^2\pi^2}{L^2}\right)\dot{V}_z(n,t)A_d$$
$$+\frac{S_0}{\mu_z}\left(4B_a(n,k)-3B_b(n,k)+B_c(n,k)\right)V_z(n,t)=\frac{1}{\mu_z}\left[Q_1 sin\frac{n\pi ct}{L}+Q_2\sin n\pi\frac{(ct+d)}{L}\right] \quad (7b)$$

Where

$$A_a=\int_0^L sin\frac{n\pi x}{L}sin\frac{k\pi x}{L}dx,\quad A_b=A_c=A_a,\quad A_d=\int_0^L cos\frac{n\pi x}{L}sin\frac{k\pi x}{L}dx$$
$$B_a(n,k)=\int_0^L x sin\frac{n\pi x}{L}sin\frac{k\pi x}{L}dx,\quad B_b(n,k)=\int_0^L x^2 sin\frac{n\pi x}{L}sin\frac{k\pi x}{L}dx,$$

$$B_c(n,k) = \int_0^L x^3 sin\frac{n\pi x}{L} sin\frac{k\pi x}{L} dx \tag{7c}$$

Further simplification of equation (7b) gives

$$\ddot{V}_z(n,t) + H_{n1}\dot{V}_z(n,t) + H_{n2}V_z(n,t) = H_{n3} sin\phi t + H_{n4} sin(\phi+\theta)t \tag{8}$$

Where

$$H_{n1} = 2R_z^0\left(\frac{n^2\pi^2}{L}\right), \quad H_{n2} = 2N\frac{n^2\pi^2}{L} + \frac{So}{\mu_2}(4x - 3x^2 + x^3)$$

$$H_{n3} = \frac{2Q_2}{\mu_2}, \quad H_{n4} = \frac{2Q_1}{\mu_2}$$

$$\phi = \frac{n\pi ct}{L}, \quad \theta = \frac{n\pi d}{L} \tag{9}$$

Subjecting equation (8) to Laplace transform and after simplification yields

$$V_z(s)[s^2 + H_{n1}s + H_{n2}] = H_{n3}\left(\frac{\phi}{s^2+\phi^2}\right) + H_{n4}\left(\frac{\phi}{s^2+\phi^2}\right) + H_{n5}\left(\frac{s}{s^2+\phi^2}\right) \tag{10}$$

Where

$$H_{n5} = H_{n4} sin\theta \tag{11}$$

Furthermore, equation (10) becomes

$$V_z(s) = \left(H_{n3}\left(\frac{\phi}{s^2+\phi^2}\right) + H_{n4}\left(\frac{\phi}{s^2+\phi^2}\right) + H_{n5}\left(\frac{s}{s^2+\phi^2}\right)\right)\cdot\frac{1}{(s-\Omega_1)(s-\Omega_2)} \tag{12}$$

Where

$$\Omega_1 = \frac{-H_{n1} + \sqrt{H_{n1}^2 - 4H_{n2}}}{2}, \quad \Omega_2 = \frac{-H_{n1} - \sqrt{H_{n1}^2 - 4H_{n2}}}{2} \tag{13}$$

and equation (12) gives

$$\begin{aligned} V_z(s) = {} & \frac{H_{n3}}{\Omega_2 - \Omega_1}\left(\frac{\phi}{s^2+\phi^2}\cdot\frac{1}{(s-\Omega_2)} - \frac{\phi}{s^2+\phi^2}\cdot\frac{1}{(s-\Omega_1)}\right) \\ & + \frac{H_{n4}}{\Omega_2 - \Omega_1}\left(\frac{\phi}{s^2+\phi^2}\cdot\frac{1}{(s-\Omega_2)} - \frac{\phi}{s^2+\phi^2}\cdot\frac{1}{(s-\Omega_1)}\right) \\ & + \frac{H_{n5}}{\Omega_2 - \Omega_1}\left(\frac{s}{s^2+\phi^2}\cdot\frac{1}{(s-\Omega_2)} - \frac{s}{s^2+\phi^2}\cdot\frac{1}{(s-\Omega_1)}\right) \end{aligned} \tag{14}$$

The Laplace inversion of equation (14) gives

$$V_z(t)=\frac{H_{n3}}{\Omega_2-\Omega_1}(I_1-I_2)+\frac{H_{n4}}{\Omega_2-\Omega_1}(I_3-I_2)+\frac{H_{n5}}{\Omega_2-\Omega_1}(I_5-I_6) \qquad (15)$$

Where

$$I_1=e^{\Omega_2 t}\int_0^t e^{-\Omega_2 u} sin\phi u du \quad , \quad I_2=e^{\Omega_1 t}\int_0^t e^{-\Omega_1 u} sin\phi u du$$
$$I_1=I_3 \quad , \quad I_4=I_2$$
$$I_5=e^{\Omega_2 t}\int_0^t e^{-\Omega_2 u} cos\phi u du \quad , \quad I_6=e^{\Omega_1 t}\int_0^t e^{-\Omega_1 u} cos\phi u du \qquad (16)$$

The solutions to the integrals in equation (16) are given and with further rearrangements equation (15) becomes

$$V_z(t)=\frac{H_{n3}}{\Omega_2-\Omega_1}\left(\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_2^2}\cdot\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_1^2}\right)$$
$$+\frac{H_{n4}}{\Omega_2-\Omega_1}\left(\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_2^2}\cdot\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_1^2}\right)$$
$$+\frac{H_{n5}}{\Omega_2-\Omega_1}\left(\frac{A_0 sin\phi t-A_1 cos\phi t}{\phi^2+\Omega_2^2}\cdot\frac{A_0 sin\phi t-A_1 cos\phi t}{\phi^2+\Omega_1^2}\right) \qquad (17)$$

Where

$$A_0=\phi e^{-\Omega_2 t} \text{ and } A_1=\Omega_2 e^{-\Omega_2 t} \qquad (18)$$

when equation (18) is inverted, one obtains

$$W_z(x,t)=\frac{1}{\Omega_2-\Omega_1}\sum_{n=1}^{\infty}\left[H_{n3}\left(\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_2^2}\cdot\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_1^2}\right)\right.$$
$$+H_{n4}\left(\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_2^2}\cdot\frac{A_0 cos\phi t-A_1 sin\phi t}{\phi^2+\Omega_1^2}\right)$$
$$\left.+H_{n5}\left(\frac{A_0 sin\phi t-A_1 cos\phi t}{\phi^2+\Omega_2^2}\cdot\frac{A_0 sin\phi t-A_1 cos\phi t}{\phi^2+\Omega_1^2}\right)\right]\times\frac{sin n\pi x}{L} \qquad (19)$$

Equation (19) is the transverse deflection of uniform Rayleigh beam with an accelerating distributed mass on variable elastic foundation at uniform speed.

4. DISCUSSION AND RESULTS

This section investigates the dependence of the Rayleigh beam response on variable elastic foundation, rotatory inertia correction factor, axial force, distance and load parameters at uniform speed. Making use of the theoretical analysis described in the previous sections and in order to carry out the calculations of practical interest in dynamics of structure and engineering designs, a thick beam of length 12.92m is considered for illustrative examples. It is assumed that the mass travels at the constant velocity 3.128m/s, the modulus of elasticity $E=2.109\times10^9$, the moment of inertia $I=2.37698\times10^{-3}$, and the mass per unit length of the beam is 4501.537g/m.

The dynamic displacement responses of uniform Rayleigh beam with an accelerating distributed mass on a variable elastic foundation are calculated and plotted against time for various values of rotatory inertia correction factor, axial force, foundation modulus, distance and loads parameters at constant speed.

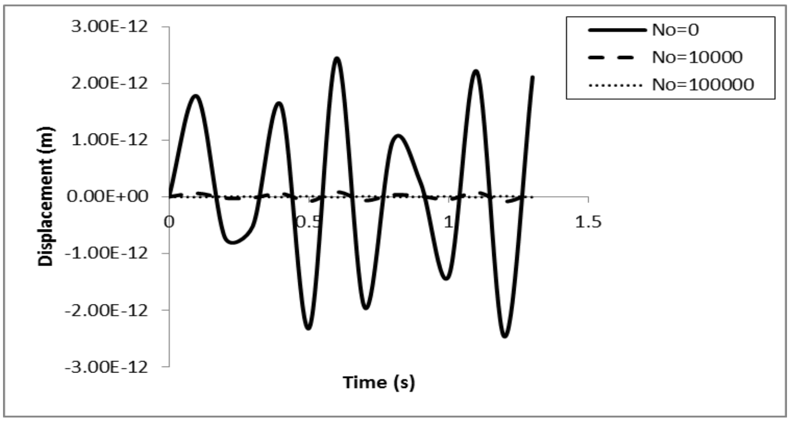

Figure 1: Deflection profile of uniform Rayleigh beam with an accelerating distributed mass on a variable foundation for fixed values of R_0, S_0, Q_1, Q_2, d and various value of N_0.

Figure 1 shows the effect of foundation modulus So on the deflection profile of uniform Rayleigh beam with an accelerating mass at constant speed. The graph show that the response amplitude decrease as the value of No increase at fixed value of Ro, So, d, Q1 and Q2.

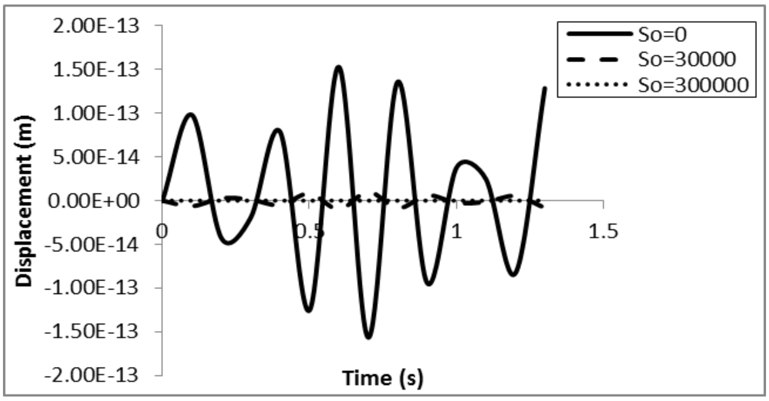

Figure 2: Deflection profile of uniform Rayleigh beam with an accelerating distributed mass on a variable foundation for fixed values of R_0, N_0, Q_1, Q_2, d and various value of S_0.

Figure 2 depicts the axial force influence on the dynamic response of uniform Rayleigh beam resting on variable elastic foundation for fixed values of rotatory inertia correction factor, axial force, distance and loads parameters. The graph shows that the transverse displacement response decreases as the foundation modulus increases.

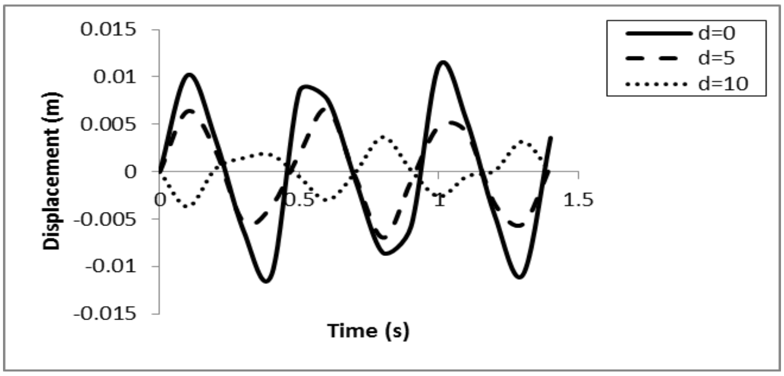

Figure 3: Deflection profile of uniform Rayleigh beam with an accelerating distributed mass on a variable foundation for fixed values of R_0, N_0, Q_1, Q_2, S_0 and various value of d.

Figure 3 depicts the effect of distance parameter on uniform Rayleigh beam for fixed values of rotatory inertia correction factor, foundation modulus, axial force and loads parameters. The graph shows that an increase in distance parameter results in the decrease of the dynamic deflection.

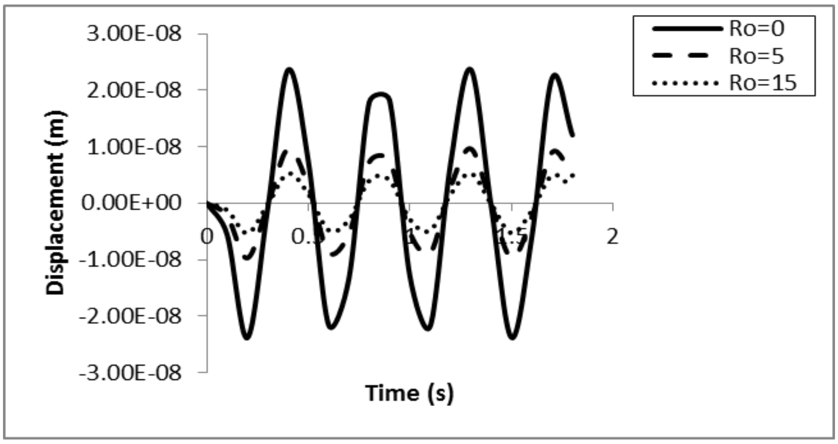

Figure 4: Deflection profile of uniform Rayleigh beam with an accelerating distributed mass on a variable foundation for fixed values of d , N_0 , Q_1 , Q_2 , S_0 and various value of R_0 .

The dynamic response of uniform Rayleigh beam on variable elastic foundation with an accelerating distributed mass at uniform speed for fixed values of So, No, d, Q1 and Q2 decreases as the rotatory inertia correction factor increases as given in figure 4.

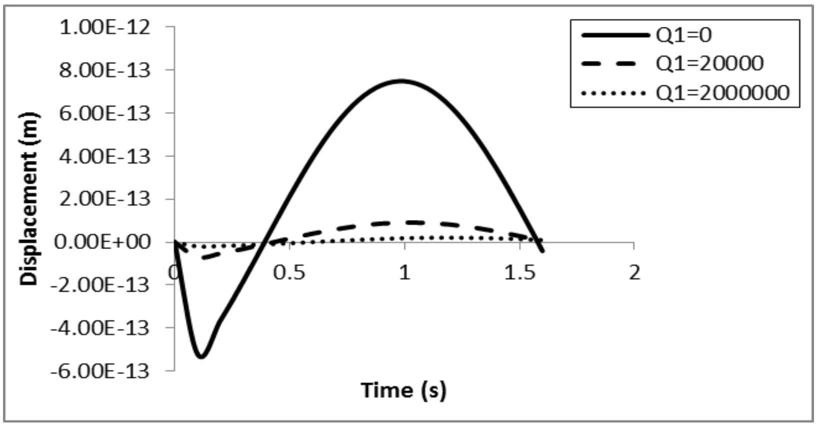

Figure 5: Deflection profile of uniform Rayleigh beam with an accelerating distributed mass on a variable foundation for fixed values of d , N_0 , R_0 , Q_2 , S_0 and various value of Q_1 .

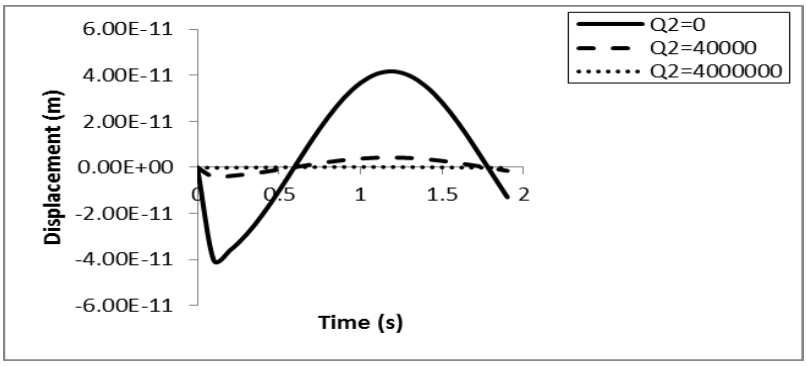

Figure 6: Deflection profile of uniform Rayleigh beam with an accelerating distributed mass on a variable foundation for fixed values of d , N_0 , R_0 , Q_1 , S_0 and various value of Q_2 .

Figures 5 and 6 shows the effect of two loads parameters on the uniform Rayleigh beam resting on variable elastic foundation at constant speed. The graph shows that displacement response decreases as the load parameters increases.

5. CONCLUSION

In this investigation, the uniform forced transverse vibration of Rayleigh beam with an accelerating distributed mass resting on a variable elastic foundation at constant velocity is presented. The solution technique is based on Fourier Sine Integral transformation, the expansion of Heaviside function in series form and convolution theory. Numerical example is given in order to determine the effects of various parameters.

The effect of axial force on the dynamic response of the beam is examined, it is observed that the dynamic deflection of the beam increases with an increment in the prestress values; the same results are also obtained for other parameters like foundation modulus, rotatory inertial correction factor and distance and load parameter.

6. REFERENCES

1. Stokes, G.G Discussion of differential equation relating to the breaking of railway bridges. Mathematical and Physical papers. Vol. 2, 178-220, 1883. https://en.wikipedia.org/wiki/Sir_George_Stokes,_1st_Baronet
2. Lee, H.P and Ng, T.Y. Transverse Vibration of a Plate moving over Multiple points supports. *Applied Acoustics*. 1996; 47(4):291-301. http://www.sciencedirect.com/science/article/pii/0003682X9500058H
3. Fryba, L.A. Vibration of solids and structures under moving loads. Thomas Telford, 1999; London. http://www.springer.com/us/book/9789401196871
4. M.H Kargarnovin and D. Younesian. dynamics of timoshenko beams on pasternak foundation under moving load. *Mechanics Research communication*. 2004;31:713-723. www.iust.ac.ir/files/rail/Abstract/younesian_2004_1.pdf?origin=publication_detail
5. Oni S. T. and Ogunyebi S. N. Dynamical Analysis of finite prestressed Bernoulli Euler beams with general boundary conditions under travelling distributed loads. *Journal of the Nigerian Association of Mathematical Physics*. 2008;12:87-102. http://www.mts.futa.edu.ng/profile_publication.php?staffid=528
6. Volterra, E, and Zackmanoglu, E.C. Dynamics of vibration. Charles E. Merrill Books. 1965. http://www.worldcat.org/title/dynamics-of-vibrations/oclc/499038573
7. Inglis C.E. A mathematical treatise on vibration in Railway Bridges. The University press, Cambrige. 1934. https://search.yahoo.com/yhs/search?p=Inglis+C.E.+A+mathematical+treatise+on+vibration+in+Railway+Bridges&ei=UTF-8&hspart=mozilla&hsimp=yhs-002
8. Shahin N.A and Mbakisya. Simply supported beam response on elastic foundation carrying repeated rolling concentrated loads. *Journal of Engineeering Sciece and Technology*. 2010;5(1):52-68. https://search.yahoo.com/yhs/search?p=Inglis+C.E.+A+mathematical+treatise+on+vibration+in+Railway+Bridges&ei=UTF-8&hspart=mozilla&hsimp=yhs-002
9. Sadiku, S and Leipholz, H.H.E. On the Dynamics of elastic systems with moving concentrated masses. *Ingenieur Archive*. 1981;57:223-242. www.scielo.br/scieloOrg/php/articleXML.php?pid=S1679-78252010000300004
10. Gbadeyan, J.A. and Oni, S.T. Dynamic Behaviour of Beams and Rectangular Plates under moving loads. *Journal of Sound and Vibration*. 1996;182(5):677-695. https://www.deepdyve.com/lp/elsevier/dynamic-behaviour-of-beams
11. Stanisic, M. M, Hardin, J.A and Lou, Y.C. on the response of the plate to a multi-massaving system. *Acta mechanical*. 1968;5:37-53. https://www.researchgate.net/publication/244996692_On_the_response_of_the_plate_to_a_multi-masses_moving_system
12. E. Esmailzadeh and M. Gorashi. Vibration analysis of beams traversed by uniform partially distributed moving masses. *Journal of Sound and Vibration*. 1995;184:9-17. www.scirp.org/reference/ReferencesPapers.aspx?ReferenceID=41552
13. J. A. Gbadeyan and M.S. Dada dynamic response of plates on pasternak foundation to distributed moving loads. *Journal of Nigerian Association of Mathematical Physics.* 2001;5:184-200. www.eajournals.org/wp-content/uploads/DYNAMIC.
14. Gbadeyan, J.A. and Dada, M.S. The effects of linearly varying distributed moving loads on Beams. *Journal of Engineering and Applied Sciences*. : 2007;2(6): 1006-1011. https://www.medwelljournals.com/abstract/?doi=jeasci.2007.1006.1011
15. Rieker J.R and Trethewey M.W. Finite Element Anaalysis of an Elastis Beam Structure subjected to a moving distributed mass train. *Mechanical System and signal processing*. 1999;13:31-51. http://adsabs.harvard.edu/abs/1999MSSP...13...31R
16. Usman M.A, Hammed F.A, and Onitilo S.A. Vibration of Timoshenko beam subjected to partially distributed moving load. *Journal of Scientific Research and Studies* 2015; 2(4):80-86. *www.modernrespub.org/jsrs/pdf/2015/June/Usman et al.pdf*
17. Strutt J.W. Theory of Sound. London: Macmillan publication Co., Inc. 1877. https://www.abebooks.co.uk/servlet/BookDetailsPL?bi=15560103674&searchurl=tn%3Dtheory%2Bof%2Bsound%26sortby%3D17%26an%3Drayleigh

Cite this article: Ogunyebi Segun Nathaniel and Adedowole Alimi. ON THE DYNAMIC BEHAVIOR OF UNIFORM RAYLEIGH BEAM WITH AN ACCELERATING DISTRIBUTED MASS. *Am. J. innov. res. appl. sci.* 2017; 4(4): 136-143.