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ORIGINAL ARTICLE

# INFLUENCE OF EXPONENTIALLY DECAYING FOUNDATION ON THE RESPONSE OF NON-UNIFORM BEAMS UNDER UNIFORMLY DISTRIBUTED LOAD

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## ABSTRACT

**Background:** Flexural vibration of structural elements subjected to moving loads is a topic that attracts the attentions of researchers in field of Engineering and Mathematical Physics. **Objectives:** To obtain analytical solutions of the governing fourth order partial differential equation and establish the resonance conditions for moving distributed loads. **Methods**: The technique is based on the generalized Galerkin's method and integral transform. **Results:** Analyses show that the higher values of the foundation modulli decrease the transverse deflections of the non-uniform Bernoulli-Fuler beam. **Conclusions:** The analytical solution for the non uniform is solved, the effect of exponential decaying foundation and resonance condition are determined.
***Keywords**: exponentially decaying foundation, non-uniform beam, distributed loads, vibrating system, differential equation.*

## 1. INTRODUCTION

Several innovations and researches brought about by technology advancement have widened the scope of application of structural members subjected to loads (static or dynamic). Common examples of such structures include; beam, plates and shells while moving loads include moving trains, trucks, cars, bicycles cranes and so on. The study is very paramount on our day to day activities and has attracted and continues to attract the attentions of researchers in Mechanical, Civil, Aerospace Engineering, and Mathematical Physics and so on.

Among the earliest work in this area of study is the work of Stokes (1849) who obtained an approximate solution for the response of a beam by neglecting the mass of the beam [3]. In particular, the dynamic response of a simply supported beam transverse by a constant force moving at a uniform speed was first studied by Krylov (1905) [4]. He used the method of expansion of Eigen function to obtain his results. Lowan (1935) also considered the problem of transverse oscillations of beams under the action of moving loads for the general case of any arbitrarily prescribed law of motion [5]. He obtained his solution using Green's functions. More recently, Omolofe et al (2011) investigated exact vibration solution for initially stressed beams resting on elastic foundation subjected to partially distributed masses. A procedure involving spectra Galerkin's method, Integral transformation and convolution theory is employed to investigate the transverse behaviour of a thin structural member supported with elastic foundation having exponential stiffness [6]. A closed form solution of the fourth order partial differential equations describing the motion of the dynamical system was obtained. Omolofe and Ogunyebi (2016) studied the dynamic behaviour of a rotating Timoshenko beam when under the actions of a variable magnitude load moving at non-uniform speed [7]. The effect of cross-sectional dimension and damping on the flexural motions of the elastic beam was neglected. The coupled second order partial differential equations incorporating the effects of rotary and gyroscopic moment describing the motions of the beam was scrutinized in order to obtain the expression for the dynamic deflection and rotation of the vibrating system using an elegant technique called Galerkin's Method. It was found that the response amplitude of the simply supported beam increases with an increase in the value of the foundation reaction modulus. These works, though impressive, were based on the vibration analysis of an Elastic beam with uniform cross-section. In particular, both moment of inertia $I$ and mass per unit length $\mu$ of the beam did not vary with spartial coordinate $x$ along the span of the beam.

Among the works on non uniform structure is also the work of Oni (1996) who considered the response of a non uniform thin beam resting on a constant elastic foundation to several moving masses [8]. For the solution of the problem, he used the versatile technique of Galerkin to reduce the complex governing fourth order partial differential equation with variable and singular coefficients to a set of ordinary differential equations. The set of ordinary differential equations was later simplified and solved using modified asymptotic method of Struble. Other studies on non-uniform beam include

Douglas et al (2002) who considered the cantilever strip plate of varying thickness and the centre of shear [9]. Oni and Awodola (2003) investigated vibrations under a moving load of a non-uniform Rayleigh beam on variable elastic foundation and Oni and Omolofe (2005) studied dynamic behaviour of non-uniform Bernoulli-Euler beams subjected to concentrated loads traveling at varying velocities [10,11].

Recently, Mohamed and Abohadima (2008) investigated Mathematical model for vibrations of non-uniform flexural beams [12]. They solved dynamic equation of the beam by introducing new variables to transform the equation to the Bessel differential equations. The obtained solutions are used to find the mode shapes and the natural frequencies. The fundamental natural frequency decreases as the slenderness ratio increases and as the non uniformity factor decreases. Ojih et al (2013) investigated the dynamic response of non uniform Rayleigh beam resting on Pasternak foundation and subjected to concentrated loads travelling at constant velocity with simply supported boundary condition [13]. The results show that as the Rotatory inertia increases, the response amplitudes of the non uniform Rayleigh beam decreases for both moving force and moving mass problems.

Very recently, Adedowole (2016) worked on flexural motions under moving distributed masses of Beam- type structures on Vlasor foundation and having time dependent boundary conditions [14]. He made used of Mindlin and Goodman's technique to transform the governing non-homogenous fourth order partial differential equations with non- homogenous boundary conditions into non-homogenous fourth order partial differential equation with homogenous boundary conditions. The author also consider flexural vibration of non prismatic Rayleigh beam with non uniform prestress under concentrated loads moving with variable velocity [15].

Thus, an analytical approach is developed to assess the dynamic response behaviour of non uniform beams resting on exponentially decaying foundation and subjected to travelling loads in this paper. Several numerical examples will also be presented to classify the influence of exponentially decaying foundation on the dynamic response of the beams to moving loads. It is assumed that the speed at which the travelling load traverses the structural elements is constant.

## 2. Definition of the Problem

Assuming non-uniform simply supported beam with length L under distributed load. The distributed loads $M_i$ move along the beam starting at time t = 0 with constant velocity $C_i$. The equation of motion for the system is given by the fourth order partial differential equation Adedowole (2016).

$$-\frac{\partial Q(x,t)}{\partial x}+\mu(x)\frac{\partial^2 y(x,t)}{\partial t^2}-N\frac{\partial^2 y(x,t)}{\partial x^2}+b(x)\frac{\partial y(x,t)}{\partial t}+F(x)y(x,t)-q(x,t)=0 \qquad (1)$$

$$Q(x,t)=\frac{\partial D(x,t)}{\partial x} \qquad (2)$$

where $Q(x,t)$ is the shear force, $q(x,t)$ is the constant moving distributed force acting on the beam, $\mu$ is the mass of the beam per unit length L,b is the material damping intensity, $y(x,t)$ is the vertical response of the beam, $D(x,t)$ is the flexural moment and t is time.
The flexural moment acting on the beam across section is related to the vertical response as

$$D(x,t)=-EI(x)\frac{\partial^2 y(x,t)}{\partial x^2} \qquad (3)$$

Where $EI(x)$ the flexural rigidity of the beam, E is is the young modulus
The distribution of the non-uniform characteristics may be assumed as power functions Mohamed (2008). The parameters $\alpha$ and $n$, are used to approximate the actual non-uniformity of the beam given as

$$I(x)=I_o(1+\alpha x)^{n+2} \qquad (4a)$$
$$\mu(x)=\mu_o(1+\alpha x)^n \qquad (4b)$$
$$b(x)=b_o(1+\alpha x)^n \qquad (4c)$$

where $I(x)$ is the variable moment of inertia of the be beam, $I_o$, $\mu_o$ and $b_o$ are the beam characteristics at $x=0$.

**3. The boundary conditions:** The boundary conditions depend on the constraints at the beam ends, however for a simply supported beam whose length is L, the vertical displacement at the beam ends are given as:

$$y(0,t) = y(L,t) = 0 \tag{5a}$$

$$y''(0,t) = y''(L,t) = 0 \tag{5b}$$

Where dash means derivative with respect to $x$
It is assumed that the initial conditions are

$$y(x,0) = 0 = \frac{\partial^2 y(x,0)}{\partial t^2} \tag{6}$$

**4. CASE 1: Analysis of non-uniform beam with exponentially decaying foundation to constant magnitude moving distributed forces:** In this paper, we adopt the example in Omolofe (2011) and define the exponential decaying elastic foundation $F(x)$ as:

$$F(x) = F_o e^{-\lambda x} \tag{7}$$

where $\lambda$ is a constant and $F_o$ is the elastic foundation constant
Furthermore, the constant vertical excitation acting on the beam is chosen as

$$q(x,t) = PH(x - c_i t) \tag{8}$$

The distributed load is assumed to be of mass M and the time t is assumed to be limited to that interval of time within the mass on the beam, that is;

$$0 \le f(t) \le L \tag{9}$$

and $H(x-ct)$ is the Heaviside function defined as;

$$H(x-ct) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases} \tag{10a}$$

with the properties,

i. $$\frac{d}{dx}\{H(x-ct)\} = \delta(x-ct)$$

ii.

iii. $$H(x-ct)f(x) = \begin{cases} 0, & x \le ct \\ f(x), & x \ge ct \end{cases} \tag{10b}$$

where $\delta(x-ct)$ represents the Dirac delta function and $H(x-ct)$ is a typical engineering function made to measure engineering applications which often involved functions that are either "off" or "on". $c_i$ is the velocity of the i[th] particle of the system, t is the travelling time substituting equations (2), (3), (4), (5), (6) & (7) into equation (1)
Taking n = 1 for simplicity yields.

$$EI_o(x)\left[\frac{\partial^2}{\partial x^2}\left((1+\alpha x)^3 \frac{\partial^2 y(x,t)}{\partial x^2}\right)\right] + \mu_o(1+\alpha x)\frac{\partial^2 y(x,t)}{\partial t^2}$$
$$+ b_0(1+\alpha x)\frac{\partial y(x,t)}{\partial t} + F_0 e^{-\lambda x} y(x,t) = PH(x - c_i t) \tag{11}$$

To the authors best of knowledge, a closed form solution to the fourth order Partial Deferential Equation (1) governing the motion of the slender beam under the actions of moving force, does not exist. It is desirable to obtain some vital information about the vibrating system.

**5. approximate analytical solution:** To solve the beam problem above in equation (11), we shall use the versatile solution technique called Galerkin's method often used in solving diverse problems involving mechanical vibrations Ojih (2013). The equation of the motion of an element of the beam is generally symbolically written in the form.

$$\Gamma y(x,t) - q(x,t) = 0 \tag{12}$$

where $\Gamma$ is the differential operator with variable coefficients, y(x,t) is the beam displacement, q(x,t) is the load acting on the beam, x and t are spatial coordinates and time respectively. The solutions of the system of equation (11) is expressed as

$$y(x,t) = \sum_{i=1}^{n} W_i(t) Q_i(x) \tag{13}$$

where $W_i(t)$ are coordinates in modal space and $Q_i(x)$ are the normal modes of free vibration written as

$$Q_i(x) = \sin\theta_i x + A_i \cos\theta_i x + B_i \sinh\theta_i x + C_i \cosh\theta_i x \tag{14}$$

where the constant, $A_i, B_i \quad and\, C_i$ define the space and amplitude of the beam vibration. Their values depend on the boundary condition associated with the structure. Thus, for a simply supported beam, it can be shown that

$$A_i = B_i = C_i = 0 \text{ and } \quad \theta_i = \frac{i\pi}{L} \tag{15}$$

Thus, for a beam with simple supports at both ends, equation (14) takes the form

$$Q_i(x) = \sin\frac{i\pi x}{L} \tag{16}$$

Thus in view of equation (16) the transverse displacement response of a simply supported elastic beam, using an assumed mode method can be written as

$$y(x,t) = \sum_{i=1}^{n} W_i(t) \sin\frac{i\pi x}{L} \tag{17}$$

Substituting (17) into the governing (11) and after some simplifications and arrangements, one obtains

$$\begin{aligned}
&EI_o(x)\left[\frac{\partial^2}{\partial x^2}\left(\left(1+3\alpha x+3\alpha^2x^2+\alpha^3x^3\right)\frac{\partial^4}{\partial x^4}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}\right)\right.\\
&+\left(3\alpha+6\alpha^2x+3\alpha^3x^2\right)\frac{\partial^3}{\partial x^3}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}+\left(6\alpha^2+6\alpha^3x\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}+\\
&\left.+(1+\alpha x)\frac{\partial}{\partial x}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}\right]+\mu_o(1+\alpha x)\frac{\partial^2 y(x,t)}{\partial t^2}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}\\
&+b_0(1+\alpha x)\frac{\partial}{\partial t}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}+F_0e^{-\lambda x}\sum_{i=1}^{n} W_i(t)\sin\frac{i\pi x}{L}=PH(x-c_it)
\end{aligned} \tag{18}$$

To determine, $W_i$ the expressions on the left hand sides of equation (18) are required to be orthogonal to the function $\sin\frac{k\pi x}{L}$. Thus,

$$\int_0^L \sum_{i=1}^{n}\left[ EI_o\left\{ G_1(x)\left(\frac{i\pi}{L}\right)^4 \sin\frac{i\pi x}{L} - G_2(x)\left(\frac{i\pi}{L}\right)^3 \cos\frac{i\pi x}{L} - G_3(x)\left(\frac{i\pi}{L}\right)^2 \sin\frac{i\pi x}{L} \right\} W_i(t) \right.$$
$$\left. + b_0\dot{W}_i(t)\sin\frac{i\pi x}{L} + \mu_o G_4(x)\ddot{W}_i(t)\sin\frac{i\pi x}{L} + F_0 e^{-\lambda x} W_i(t)\sin\frac{i\pi x}{L} \right] \sin\frac{k\pi x}{L}$$
$$= \int_0^L PH(x - c_i t)x\sin\frac{k\pi x}{L}dx \qquad (19)$$

Where

$$G_1(x) = \left(1 + 3\alpha x + 3\alpha^2 x^2 + \alpha^3 x^3\right) \qquad (20)$$
$$G_2(x) = \left(3\alpha + 6\alpha^2 x + 3\alpha^3 x^2\right) \qquad (21)$$
$$G_3(x) = \left(6\alpha^2 + 6\alpha^3 x\right) \qquad (22)$$
$$G_4(x) = (1 + \alpha x) \qquad (23)$$

Further rearrangements and simplifications of equation (19) we obtain

$$a_1(i,k)\ddot{W}_i(t) + a_2(i,k)\dot{W}_i(t) + a_3(i,k)W_i(t) = -P\left(\frac{k\pi}{L}\right)\cos\frac{k\pi ct}{L} \qquad (24)$$

where

$$a_3(i,k) = EI_o[H_1 - H_2 - H_3] + H_4 \qquad (25)$$

$$a_1(i,k) = \mu_o[I_1 + \alpha I_2] \qquad (26)$$

$$a_2(r,k) = b_o\frac{I_1}{2} \qquad (27)$$

$$H_1 = \left(\frac{r\Pi}{L}\right)^4\left[I_1 + 3\alpha I_2 + 3\alpha^2 I_3 + \alpha^3 I_4\right] \qquad (28)$$

$$H_2 = \left(\frac{r\Pi}{L}\right)^3\left[3\alpha I_5 + 6\alpha^2 I_6 + 3\alpha^3 I_7\right] \qquad (29)$$

$$H_3 = \left(\frac{r\Pi}{L}\right)^2\left[6\alpha^2 I_1 + 6\alpha I^3 I_2\right] \qquad (30)$$

The integrals $I_i$ are as follow

$$I_1 = \int_0^L \sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx, \qquad I_2 = \int_0^L x\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$
$$I_3 = \int_0^L x^2\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx, \quad I_4 = \int_0^L x^3\sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$
$$I_5 = \int_0^L \cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx, \qquad I_6 = \int_0^L x\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$
$$I_7 = \int_0^L x^2\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx \qquad (31)$$

Equation (24) is the second order ordinary differential equation with constant coefficient to a transformation

In what follow we subject the system of ordinary differential equation (24) to a Laplace transform defined as

$$(\sim) = \int_{\circ}^{\infty}(\cdot)e^{-st}dt \tag{32}$$

In conjunction with the initial conditions define in (8), yields the following algebraic equation

$$\left[a_1(r,k)s^2 + a_2(r,k)S + a_3(r,k)\right]W_r(s) = P_0\frac{S}{S^2+\theta^2} \tag{33}$$

Where

$$P_0 = \left(\frac{k\pi}{L}\right), \quad \theta = \left(\frac{k\pi ct}{L}\right) \tag{34}$$

Subjecting equation (33) for further simplification yields

$$W_i(S) = \frac{P_o}{(d_1-d_2)}\left(\frac{S}{S^2+\theta^2}\cdot\frac{1}{S-d_1} - \frac{S}{S^2+\theta^2}\cdot\frac{1}{S-d_2}\right) \tag{35}$$

Where

$$d_1 = \frac{-a_2+\sqrt{a_2^2-4a_1a_3}}{2a_1} \tag{36}$$

$$d_2 = \frac{-a_2-\sqrt{a_2^2-4a_1a_3}}{2a_1} \tag{37}$$

To obtain the Laplace inversion of equation (35), we shall adopt the following representations

$$g(s) = \frac{S}{S^2+\theta^2},\ f_1(s) = \frac{1}{S-d_1} \text{ and } f_2(s) = \frac{1}{S-d_2} \tag{38}$$

So that the Laplace inversion of each term of the RHS (35) is the convolution of $f_i$'s and g defined by as

$$f_s * g = \int_0^t f_i(t-u)g(u)du, \qquad i = 1,2 \tag{39}$$

Thus the Laplace inversion of (35) is given by

$$W_i(t) = P_k\left[\frac{e^{d_1t}}{d_1}I_8 - \frac{e^{d_2t}}{d_2}I_9\right] \tag{40}$$

where

$$I_8 = \int_0^t e^{-d_1u}\cos\theta u du \quad \text{and} \quad I_9 = \int_0^t e^{-d_2u}\cos\theta u du$$

$$P_k = \frac{P_0}{(d_1-d_2)} \tag{41}$$

Evaluating integrals in equation (41) we have

$$I_8 = \frac{-\theta e^{d_1t}\sin\theta t + d_1 - d_1e^{d_1t}\cos\theta t}{(\theta^2+d_1^2)}$$

$$I_8 = \frac{-\theta e^{d_1 t}\sin\theta t + d_1 - d_1 e^{d_1 t}\cos\theta t}{(\theta^2 + d_1^2)} \qquad (42)$$

Further simplification of equation (40) yields

$$W_i(t) = P_k\left[\frac{e^{d_1 t}}{d_1(\theta^2 + d_1^2)}\left(d_1 - \theta e^{d_1 t}\sin\theta t - d_1 e^{d_1 t}\cos\theta t\right)\right.$$

$$\left. - \frac{e^{d_2 t}}{d_2(\theta^2 + d_2^2)}\left(d_2 - \theta e^{d_2 t}\sin\theta t - d_2 e^{d_2 t}\cos\theta t\right)\right] \qquad (43)$$

Substituting equation (43) into equation (13) which on inversion yields

$$y(x,t) = \sum_{i=1}^{n} P_k\left[\frac{e^{d_1 t}}{d_1(\theta^2 + d_1^2)}\left(d_1 - \theta e^{d_1 t}\sin\theta t - d_1 e^{d_1 t}\cos\theta t\right)\right.$$

$$\left. - \frac{e^{d_2 t}}{d_2(\theta^2 + d_2^2)}\left(d_2 - \theta e^{d_2 t}\sin\theta t - d_2 e^{d_2 t}\cos\theta t\right)\right]\sin\frac{i\pi x}{L} \qquad (44)$$

Equation (44) represents the transverse displacement response of the non-uniform Bernoulli beam resting on exponentially decaying foundation under the action of fast moving distributed forces

## 6. CASE II Response of non-uniform beam to Harmonic variable magnitude moving distributed loads:

The dynamic behavior of structurally damped non-uniform beam when subjected to harmonic variable magnitude moving load is investigated in this section. Thus, the load P(x,t) is given as

$$q(x,t) = P\sin\Omega t H(x - c_m t) \qquad (45)$$

where $\Omega$ is the circular frequency of the harmonic force and all parameters are as defined previously substituting equation (45) in equation (1), vibration of the beam is then described by the equation

$$-\frac{\partial Q(x,t)}{\partial x} + \mu(x)\frac{\partial^2 y(x,t)}{\partial t^2} + b(x)\frac{\partial y(x,t)}{\partial t} + F(x)y(x,t) = P\sin\Omega t H(x - c_m t) \qquad (46)$$

Equation (46) is the governing equation describing the motion of non-uniform beams subjected to last moving loads of varying magnitude. Like in the previous section, as closed-form solution to equation (46) is sought.
To this effect, use is made of an assumed mode method already alluded to and by this method the transverse detection $y_b(x,t)$ of non-uniform beam under the action of variable magnitude moving load can be written as

$$y_b(x,t) = \sum_{m=1}^{n} W_m(t) D_m(x) \qquad (47)$$

Where $W_m(t)$ coordinates in modal are space and $D_m(x)$ are the normal modes of free vibration.
Thus, for a simply supported beam equation (47) becomes

$$y_b(x,t) = \sum_{m=1}^{\infty} W_m(t)\sin\frac{m\pi x}{L} \qquad (48)$$

In view of equation (48) and following the same arguments as in the previous section and after some simplifications and rearrangements equation (46) becomes

$$\sum_{m=1}^{\infty}\left(a_1(m,k)\ddot{W}_m(t)+a_2(m,k)\dot{W}_m(t)+a_3(m,k)W_m(t)\right)=P_o\sin\Omega t\cos\frac{k\pi ct}{L} \tag{49}$$

Where $c_m$ is the velocity of the $m^{th}$ particle of the system and other parameter are as defined previously

Without loss of generality, considering only the mth particle of the dynamical system yields

$$a_1(m,k)\ddot{W}_m(t)+a_2(m,k)\dot{W}_m(t)+a_3(m,k)W_m(t)=P_o\sin\Omega t\cos\theta \tag{50}$$

Equation (50) is analogous to equation (24), thus subjection equation (50) to Laplace transform in conjunction with the boundary conditions stated (7) and using convolution theory we obtain

$$\begin{aligned}W_m(t)=P_k\Bigg[&\frac{e^{d_1t}\eta_1}{d_1(\eta_1^2+d_1^2)}\left(1-\frac{d_1}{\eta_1}e^{d_1t}\sin\eta_1t-e^{d_1t}\cos\eta_1t\right)\\&+\frac{e^{d_1t}\eta_2}{d_1(\eta_2^2+d_1^2)}\left(1-\frac{d_1}{\eta_2}e^{d_1t}\sin\eta_2t-e^{d_1t}\cos\eta_2t\right)\\&-\frac{e^{d_2t}\eta_1}{d_2(\eta_1^2+d_2^2)}\left(1-\frac{d_2}{\eta_1}e^{d_2t}\sin\eta_1t-e^{d_2t}\cos\eta_1t\right)\\&-\frac{e^{d_2t}\eta_2}{d_2(\eta_2^2+d_2^2)}\left(1-\frac{d_2}{\eta_2}e^{d_2t}\sin\eta_2t-e^{d_1t}\cos\eta_2t\right)\Bigg]\end{aligned} \tag{51}$$

Which on inversion yields

$$\begin{aligned}y_b(x,t)=\sum_{m=1}^{\infty}P_k\Bigg[&\frac{e^{d_1t}\eta_1}{d_1(\eta_1^2+d_1^2)}\left(1-\frac{d_1}{\eta_1}e^{d_1t}\sin\eta_1t-e^{d_1t}\cos\eta_1t\right)\\&+\frac{e^{d_1t}\eta_2}{d_1(\eta_2^2+d_1^2)}\left(1-\frac{d_1}{\eta_2}e^{d_1t}\sin\eta_2t-e^{d_1t}\cos\eta_2t\right)\\&-\frac{e^{d_2t}\eta_1}{d_2(\eta_1^2+d_2^2)}\left(1-\frac{d_2}{\eta_1}e^{d_2t}\sin\eta_1t-e^{d_2t}\cos\eta_1t\right)\\&-\frac{e^{d_2t}\eta_2}{d_2(\eta_2^2+d_2^2)}\left(1-\frac{d_2}{\eta_2}e^{d_2t}\sin\eta_2t-e^{d_1t}\cos\eta_2t\right)\Bigg]\sin\frac{m\pi x}{L}\end{aligned} \tag{52}$$

Equation (52) represents the transverse displacement response of non-uniform beam resting on exponentially decaying foundation under actions of harmonic variable magnitude moving distributed loads.

## 7. DISCUSSION ON THE CLOSED FORM SOLUTION

The transverse displacement of an elastic beam may increase without bound. Thus one is interested in resonance conditions. Equation (51) clearly depicts that the non-uniform beam resting on exponentially decaying foundation traverse by a constant moving load will grow without bound whenever

$$d_1=d_2,\qquad d_1^2=-\theta^2\qquad or\qquad d_2^2=-\theta^2 \tag{53}$$

and the velocity at which this occurs, known as critical velocity is given by the relation

$$c_i^2=\left(a_a\left(a_2^2-4a_1a_2\right)^{1/2}+2a_1a_3-a_2^2\,\frac{1}{2}\left(\frac{L}{a_1k\pi}\right)^2\right) \tag{54}$$

While equation (51) shows that the same beam under the action of harmonic variable magnitude moving loads will experience resonance effects whenever

$$d_1 = d_2, \qquad d_1^2 = -\eta_1^2 \qquad or \qquad d_2^2 = -\eta_1^2 \tag{55}$$

and the velocity at which this occurs, known as critical velocity is given by the relation

$$c_i^2 = \left( a_a \left( a_2^2 - 4a_1a_2 \right)^{1/2} + 2a_1a_3 - a_2^2 \frac{1}{2}\left( \frac{L}{a_1 k\pi} \right)^2 \right) \tag{56}$$

Therefore, it is evident from equations (54) and (56) that the critical velocity of non-uniform beam resting exponentially decaying foundation and under the actions of constant magnitude moving load is greater than that of the same beam under the action variable magnitude moving load. Hence, resonance is reached earlier in the latter.

## 8. REMARKS ON NUMERICAL RESULTS

In order to illustrate the foregoing analysis, the non-uniform beam of length 12.20m considered. Furthermore, flexural rigidity EI is 6.068 x$10^6$m$^3$/5$^2$, α = 0.025 and the moving load is assumed to travel at the constant velocity of 8.123m/s. The transverse deflections of the beam are calculated and plotted against time for various values of foundation constant (moduli) and spatial coordinates x. Values of $F_0$ between 0 N/m$^3$ and 500000 N/ m$^3$.

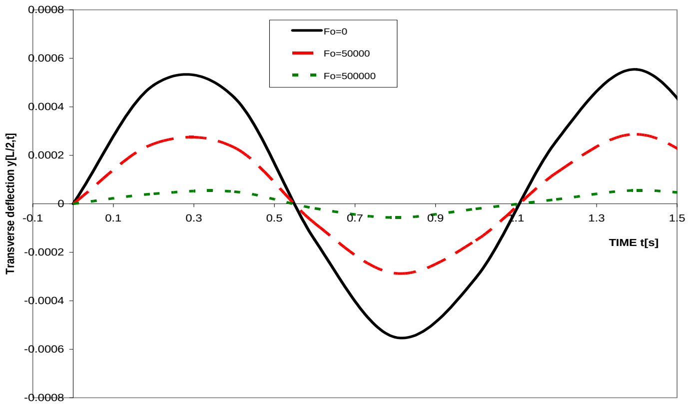


**Figure 1:** Displacement response of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of foundation modulus $F_0$.

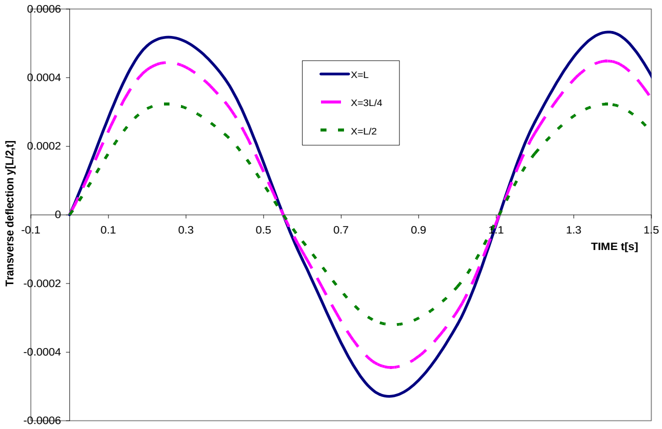


**Figure 2:** Transverse displacement response of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to constant magnitude moving loads for various values of spatial coordinates x and for fixed value of foundation modulus $F_0$ = 4000.

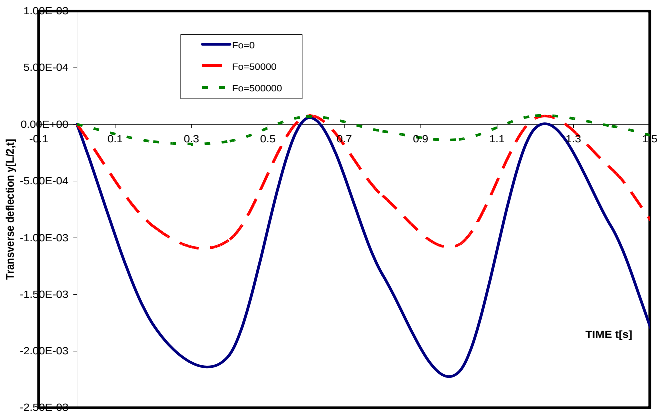


**Figure3:** Deflection profile of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to Harmonic variable magnitude moving loads for various values of foundation modulus $F_0$.

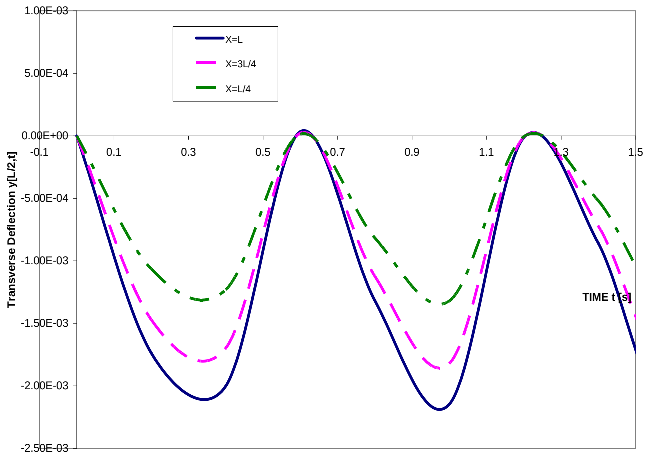


**Figure 4:** Transverse displacement response of a simply supported structurally damped thin beam resting on exponentially decaying foundation and subjected to Harmonic variable magnitude moving loads for various values of spatial coordinates and for fixed value of foundation modulus $F_0$ = 4000.

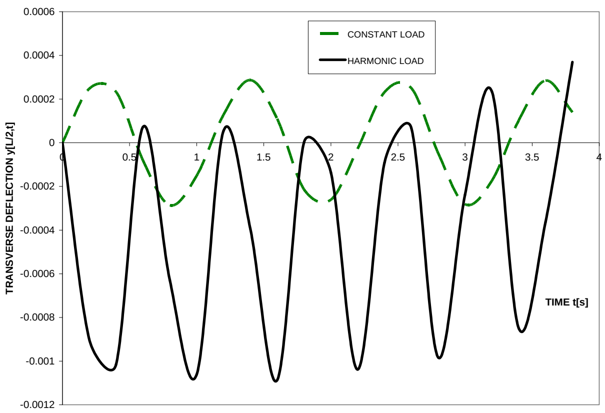


**Figure 5:**Comparison of the response of the simply supported structurally damped thin beam to constant and variable magnitude moving distributed load for foundation modulus $F_0$ =4000.

Figure 1 displays the deflection profile of the simply supported beam under the action of traveling distributed forces when traveling loads are of constant magnitude. For various values of foundation moduli $F_0$. The figure shows that as $F_0$ increases the deflection of the non-uniform beam decreases. The same results obtain when the simply supported beam is

traversed by traveling load are of variable harmonic magnitude in figure 3. Also for various time, t the displacement of the beam for fixed $F_0$. For various values of spatial coordinates x are shown in figures 2 and 4. It is deduced from these figures that the interval at which structure is supported affect the response amplitude of the beam significantly.
Finally, figure 3 depicts the comparison of the traversed displacement of constant and harmonic variable moving loads for fixed value of foundation modulus $F_0$ = 4000. Clearly, the response amplitudes of variable magnitude, moving load is higher than that of the constant magnitude moving load.

## 9 CONCLUSION

A closed form solution is presented for the displacement response of non-uniform beam under the actions of a distributed loads moving with constant velocity. The solution technique is based on Galerkin's method and integral transform. Numerical analysis is also carried out and results show the following:

**a.** For the constant and variable magnitude moving loads problems. The response amplitudes of the beam decrease with an increase in the foundation constant $F_o$.

**b.** The critical speed for the system traversed by constant moving loads is higher than that under the influence of variable moving loads.

**c.** The problem of non-uniform beam under the actions of a load moving with constant velocity, the responses amplitude of the constants load is smaller than that of the variable magnitude load. This shows that resonance is reached earlier in variable magnitude problem.

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