ORIGINAL ARTICLE

A NEW SET OF NON-STANDARD FINITE DIFFERENCE SCHEMES FOR THE CLAIRAUT EQUATIONS

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ABSTRACT

Background: This paper presents a new set of non-standard finite difference schemes for the numerical solution of non-linear Clairaut differential equation. **Objective**: The aim is to use the combination of two modeling techniques based on two non-standard modeling rules to create a qualitatively stable discrete model for the simulation of the solution to the Clairaut equation. We seek to derive finite difference schemes that are stable and correctly replicate the dynamics of the Clairaut equation. **Methods**: The method employ the use of some normalized denominator functions and non-local transformation of the derivatives base on the required properties stated in the rule 2 of the non-standard modeling rules. **Results**: We have generated discrete models whose solutions replicate the dynamics of the Clairaut equation **Conclusion**: The resulting schemes were tested and found to have the same monotonic proprepties as the Clairaut equation.

Keywords: *Clairaut Equations, Non-standard methods, Normalized denominator functions, Non-linear equation, Hybrid.*

1. INTRODUCTION.

Consider non-linearClairaut differential equations in form:

$$y(x) = x\left(\frac{dy}{dx}\right) + f\left(\frac{dy}{dx}\right) \qquad (1)$$

The study of equation (1) has acquired serious importance because of increased interest of researchers in the theory of deformable bodies. For example Clairaut equation approximates the equilibrium configuration of a fluid mass distorted by rotational or tidal waves. Attempts to solution dated back to Lanzano(1973) who established a particular solution which corresponds to certain type of density distribution [6]. Scientists have since based application of this equation of the available particular and general solution. This work is expected to provide wider latitude where suitable approximate models that can be used for the simulation of the dynamics of Clairut equation can be found.
Differential equations (linear or non-linear) may not have a complete solution that can be expressed in terms of a finite number of elementary functions. Analytical solution gives elegant solutions for these cases, but for a quite limited set of problems. That leaves many problems which must have their solutions estimated by using arithmetic operations on arrays of numbers. In such cases we seek approximate analytic solution by using various perturbation methods. However such procedures only hold for some limited ranges of the dimensionless system parameters and/or the independent variables. Thus, from arbitrary values of the system parameters only numerical integration techniques can provide accurate/ reliable numerical solution to the original differential equation [9].

Numerical methods have been at the center stage of providing acceptable and implementable models to approximating the solutions when this is the case. Some earlier researchers like (Dalquist 1978, Fatunla 1988, Lambert 1991) have achieved tremendous successes in this area [3,4,5] and the latter works of (Partidar 2005, Ushiki 1982, Yamaguti and Ushiki 1981, Mickens 1994, Angueluv and Lubuma 2000, 2003) have studied the issue of chaos and numerical instability in the use of standard numerical methods in the analysis and solution of differential equations [10,11,12,7,1,2] .

A lot of efforts have been expended by researchers in the area of numerical integration and most especially developing schemes that provide reliable approximation to differential equations and results have been obtained based on the analytic knowledge of such equations. Close examinations of differential equation, for which the exact schemes are known, shows that the denominator function generally are functions that are related to particular solutions or properties of general solution to the differential equation [7].
The article of Mickens (1994) therefore proposed computationally reliable Non-standard difference schemes that support the qualitative properties the Clairaut equation. Standard Finite Difference Schemes for ordinary differential equations exhibit a level of numerical instability [7]. The author gave valuable reasons for numerical instabilities in

some particular cases investigated. Thus, the preservation of the qualitative properties of the differential equations with respect to these schemes is of great significance. He proposed a new method of constructing discrete models whose solution have the same qualitative properties as that of its corresponding differential equations for all step size and thus eliminate the elementary numerical instabilities that can arise in finite difference methods for differential equations. The works of Mickens (2010), discussed nonstandard finite difference methods useful for the construction of discrete models of differential equations when numerical solutions are required [8]. Though the general rules for such schemes are not precisely known at the present time, several important criterions have been found. The paper also provide explanation of the significance of these criterion

In this work, we applied the rules of non-standard construction of finite difference schemes as designed by Mickens (1994, 2010) [7, 8]. The application of these rules help to generate discrete model with solutions that replicate the dynamics of the original equation .We proposed that the denominator h be replaced by a function:

$\psi(h) \rightarrow h + 0(h^2)\ as\ h \rightarrow 0$

For the first derivative and the non-linear part of the first derivative be approximated non-locally. Moreover the discretization function will be modified in such a way that the characteristic function of the discrete equation decay very fast.

2. DERIVATION OF THE DISCRETE MODEL

The general standard finite difference scheme fornon-linear first order ordinary differential equation can be derived by replacing the first order derivatives in the following manner using Non-standard modeling rules 2 and 3 [7].

$$y' \equiv \frac{(y_{k+1}-y_k)}{h} \tag{2}$$

$$y' \equiv \frac{(y_{k+1}-y_k)}{\psi}, \text{ where}, \psi(h) \rightarrow h + 0(h^2)\ as\ h \rightarrow 0 \tag{3}$$

$$y' \equiv \frac{(y_{k+1}-\beta y_k)}{\propto\psi} \tag{4}$$

$$y' \equiv \frac{(y_k-\beta y_{k-1})}{\propto\psi} \tag{5}$$

$$y' \equiv \frac{(y_k-y_{k-1})}{\propto\psi} \tag{6}$$

Where, $\psi(h) \rightarrow h + 0(h^2),\ \beta(h) \rightarrow 1\ as\ h \rightarrow 0,\ \propto \geq 2.$
Moreover grid point calculations will be approximated non-locally using any of the form :

$$\frac{(y_{k+1}+y_k)}{2} \tag{7}$$

$$y_k = \frac{(y_{k+1}+\beta y_k)}{2} \text{where}, \beta(h) \rightarrow 1\ as h \rightarrow 0 \tag{8}$$

$$y_k = \frac{(y_k+y_{k-1})^2}{4(2y_k-y_{k-1})}, 0 < |y_k - y_{k-1}| \ll \mathcal{E} as\ h \rightarrow 0 \tag{9}$$

i.e y_k and y_{k-1} are very close .
Suitable function for the above function $\psi$and β

$$\text{Include}\psi = \propto \sin(h), \sin(\propto h), \psi = \frac{(e^{\lambda h}-1)}{\lambda}, \beta = cos(h) \lambda \epsilon \mathbb{R}, \propto > 0. \tag{10}$$

2.1 Construction of New Schemes Clairaut Equation.

Consider the Clairaut differential equation given as:

$$(y')^2 + xy' - y = 0 \quad y(0) = c^2, \tag{11}$$

Where c is a constant. Theanalytical solution is $y = -\frac{x^2}{4}$or$y = cx + c^2$. (12)

Clairaut Scheme 1A.

Using the transformation Eq.(3), Eq.(6) and Eq.(7) above, the Eq.(11) can be replaced with an equivalent discrete model as follows:

$$(y')^2 \equiv \left[\frac{(y_k-y_{k-1})}{\propto\psi}\right]^2, \propto=2 \tag{13}$$

$$y' \equiv \frac{(y_{k+1}-y_k)}{\psi}, \tag{14}$$

$$y_k = \frac{(y_{k+1}+y_k)}{2}, \tag{15}$$

$y' = \frac{(y-(y')^2)}{x}$ becomes

$$\Rightarrow \frac{(y_{k+1}-y_k)}{\psi} = \frac{1}{x}\left[\frac{(y_{k+1}+y_k)}{2} - \left[\frac{(y_k-y_{k-1})}{2\psi}\right]^2\right] \tag{16}$$

$$\Rightarrow \left[x-\frac{\psi}{2}\right] y_{k+1} = \left[x+\frac{\psi}{2}\right] y_k - \psi\left[\frac{(y_k-y_{k-1})}{2\psi}\right]^2 \tag{17}$$

$$\Rightarrow y_{k+1} = \frac{\left[x+\frac{\psi}{2}\right]y_k - \psi\left[\frac{(y_k-y_{k-1})}{2\psi}\right]^2}{\left[x-\frac{\psi}{2}\right]}, \psi = \frac{(e^{\lambda h}-1)}{\lambda}, \lambda\epsilon\mathbb{R} \tag{18}$$

Clairaut Scheme 1B

Using Eq. (18) and changing the denominator function , we obtain an equivalent scheme

$$\Rightarrow y_{k+1} = \frac{\left[x+\frac{\psi}{2}\right]y_k - \psi\left[\frac{(y_k-y_{k-1})}{2\psi}\right]^2}{\left[x-\frac{\psi}{2}\right]}, \psi = \sin(\sigma h), \sigma\epsilon\mathbb{R} \tag{19}$$

We will also use non-local grid points

$$x = \frac{(x_{k+1}+x_k)}{2} = kh + \frac{h}{2}\ . \tag{20}$$

Clairaut Scheme 2A

Using the transformation equations Eq. (3), Eq. (5) and Eq. (7) above the Eq. (10) can be replaced with an equivalent discrete model as follows:

$y' = \frac{(y-(y')^2)}{x}$ become

$$\frac{(y_{k+1}-y_k)}{\psi} = \frac{1}{x}\left[\frac{(y_{k+1}+y_k)}{2} - \left[\frac{(y_k-\beta y_{k-1})}{\propto\psi}\right]^2\right], \tag{21}$$

$$\left[x-\frac{\psi}{2}\right] y_{k+1} = \left[x+\frac{\psi}{2}\right] y_k - \psi\left[\frac{(y_k-\beta y_{k-1})}{\propto\psi}\right]^2 \tag{22}$$

$$y_{k+1} = \frac{\left[x+\frac{\psi}{2}\right]y_k - \psi\left[\frac{(y_k-\beta y_{k-1})}{\propto\psi}\right]^2}{\left[x-\frac{\psi}{2}\right]},$$

$$\psi = \frac{(e^{\lambda h}-1)}{\lambda}, \lambda\epsilon\mathbb{R}, \propto\geq 2\ . \tag{23}$$

Clairaut Scheme2B

Using Eq. (23) and changing the denominator function ψ we obtain an equivalent scheme for Clairaut equation as:

$$y_{k+1} = \frac{\left[x+\frac{\psi}{2}\right]y_k - \psi\left[\frac{(y_k-\beta y_{k-1})}{\psi}\right]^2}{\left[x-\frac{\psi}{2}\right]}, \psi = \sin(h), \lambda\epsilon\mathbb{R}. \tag{24}$$

$$x = \frac{(x_{k+1}+x_k)}{2} = kh + \frac{h}{2}, x^2 = (kh)^2 + kh^2 + \frac{h^2}{4},$$

$$\psi = \sin(h),\ \beta = cos(h).$$

Clairaut Scheme 3A

Using the transformation in equations Eq. (3), Eq. (6) and Eq. (9), we obtain

$$(y')^2 \equiv \left[\frac{(y_k - y_{k-1})}{2\psi}\right]^2, \qquad (25)$$

$$y' \equiv \frac{(y_{k+1} - y_k)}{\psi}, \qquad (26)$$

$$y \equiv \frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})}, \qquad (27)$$

From (23), (24) and (25), $y' = \frac{(y-(y')^2)}{x}$ become $\frac{(y_{k+1} - y_k)}{\psi} = \frac{1}{x}\left[\frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})} - \left[\frac{(y_k - y_{k-1})}{2\psi}\right]^2\right]$, (28)

$$\Rightarrow y_{k+1} = y_k + \frac{\psi}{x}\left[\frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})} - \left[\frac{(y_k - y_{k-1})}{2\psi}\right]^2\right], \qquad (29)$$

$$\psi = \frac{(e^{\lambda h} - 1)}{\lambda}, \lambda \in \mathbb{R}.$$

ClairautScheme3B

Using equation (27) and changing the denominator function ψ

$$\Rightarrow y_{k+1} = y_k + \frac{\psi}{x}\left[\frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})} - \left[\frac{(y_k - y_{k-1})}{2\psi}\right]^2\right], \qquad (30)$$

$$\psi = \sin(h), \lambda \in \mathbb{R}, \quad x = \frac{(x_{k+1} + x_k)}{2} = kh + \frac{h}{2} \quad .$$

Clairaut Scheme 4A

Using the transformation Eq. (3), Eq. (5) and Eq. (9)above the Eq. (10) can be replaced with an equivalent discrete model as follows:

$y' = \frac{(y-(y')^2)}{x}$ become

$$\frac{(y_{k+1} - y_k)}{\psi} = \frac{1}{x}\left[\frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})} - \left[\frac{(y_k - \beta y_{k-1})}{2\psi}\right]^2\right], \qquad (31)$$

$$\Rightarrow y_{k+1} = y_k + \frac{\psi}{x}\left[\frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})} - \left[\frac{(y_k - \beta y_{k-1})}{2\psi}\right]^2\right], \quad \psi = \sin(h), \ \beta = \cos(h) \qquad (32)$$

Clairaut Scheme 4B

Using Eq. (32) and changing the denominator function ψ

$$\Rightarrow y_{k+1} = y_k + \frac{\psi}{x}\left[\frac{[(y_k + y_{k-1}]^2}{4\,(2y_k - y_{k-1})} - \left[\frac{(y_k - \beta y_{k-1})}{2\psi}\right]^2\right], \qquad (33)$$

$$\psi = \frac{(e^{\lambda h} - 1)}{\lambda}, \lambda \in \mathbb{R}, \beta = \cos(h), x = \frac{(x_{k+1} + x_k)}{2} = kh + \frac{h}{2}.$$

3. NUMERICAL EXPERIMENT AND RESULTS

This section deals with the numerical experiments and results. The derived schemes have been tested in comparison with some analytic solution of the Clairaut equation and the results are presented below:

Table 1: The table presents the result of Schemes 1 and 2 for h=0.01 compared with $y = -\frac{x^2}{4}$.

T	S1A	Y	S2A	S2B	ERROR S1A	ERROR S2A	ERROR S2B
1	-0.25	-0.25	-0.25	-0.25	0	0	0
1.01	-0.25505	-0.25505	-0.25505	-0.25505	0	0	0
1.02	-0.2601071	-0.2601	-0.260106	-0.260114	7.15300E-06	6.05000E-06	1.39780E-05
1.03	-0.2652086	-0.265225	-0.2652117	-0.2652196	1.63600E-05	1.33210E-05	5.36400E-06
1.04	-0.2703593	-0.2704	-0.2703672	-0.2703752	4.06800E-05	3.27530E-05	2.47960E-05
1.05	-0.2755598	-0.275625	-0.2755727	-0.2755808	6.51800E-05	5.22440E-05	4.41970E-05
1.06	-0.2808101	-0.2809	-0.2808282	-0.2808363	8.98500E-05	7.18240E-05	6.37180E-05
1.07	-0.2861104	-0.286225	-0.2861335	-0.2861417	1.14700E-04	9.15230E-05	8.32980E-05
1.08	-0.2914605	-0.2916	-0.2914888	-0.2914971	1.39500E-04	1.11223E-04	1.02937E-04
1.09	-0.2968606	-0.297025	-0.296894	-0.2969024	1.64400E-04	1.30981E-04	1.22607E-04
1.1	-0.3023107	-0.3025	-0.3023493	-0.3023577	1.89400E-04	1.50741E-04	1.42307E-04
1.11	-0.3078106	-0.308025	-0.3078544	-0.307863	2.14400E-04	1.70558E-04	1.62035E-04
1.12	-0.3133605	-0.3136	-0.3134096	-0.3134182	2.39600E-04	1.90407E-04	1.81824E-04
1.13	-0.3189603	-0.319225	-0.3190147	-0.3190234	2.64700E-04	2.10255E-04	2.01582E-04
1.14	-0.3246101	-0.3249	-0.3246698	-0.3246786	2.89900E-04	2.30163E-04	2.21401E-04
1.15	-0.3303098	-0.330625	-0.3303749	-0.3303838	3.15200E-04	2.50072E-04	2.41220E-04
2.9	-2.1020331	-2.1024997	-2.1021299	-2.1021533	4.66600E-04	3.69787E-04	3.46422E-04
2.91	-2.1166046	-2.1170247	-2.1166897	-2.1167133	4.20100E-04	3.34978E-04	3.11375E-04
2.92	-2.1312268	-2.1316001	-2.1313	-2.1313236	3.73400E-04	3.00169E-04	2.76566E-04
2.93	-2.1458995	-2.1462252	-2.1459608	-2.1459847	3.25700E-04	2.64406E-04	2.40564E-04
2.94	-2.1606231	-2.1609001	-2.1606722	-2.160696	2.77000E-04	2.27928E-04	2.04086E-04
2.95	-2.1753974	-2.1756251	-2.1754341	-2.1754582	2.27700E-04	1.90973E-04	1.66893E-04
2.96	-2.1902225	-2.1904001	-2.1902466	-2.1902707	1.77600E-04	1.53542E-04	1.29462E-04
2.97	-2.2050984	-2.205225	-2.2051096	-2.2051337	1.26600E-04	1.15395E-04	9.13140E-05
2.98	-2.2200251	-2.2200999	-2.2200232	-2.2200475	7.48600E-05	7.67710E-05	5.24520E-05
2.99	-2.2350025	-2.2350249	-2.2349873	-2.2350116	2.24100E-05	3.76700E-05	1.33510E-05
3	-2.2500308	-2.25	-2.2500019	-2.2500265	3.07600E-05	1.90700E-05	2.64640E-05

Scheme 1B was found to be grossly inadequate for this class of equation and so we did not include it in the results.

Table 2: r The table presents the esult of Schemes 1 and 2 for h = 0.01 compared with $y = cx + c^2$,$c = 2$

T	S1A	Y	S2A	S2B	ERR S1A	ERR S2A	ERR S2B
1	4	4	4	4	0	0	0
1.01	4.02	4.02	4.02	4.02	0	0	0
1.02	4.072442	4.0604	4.066542	4.066927	1.20420E-02	6.14214E-03	6.52695E-03
1.03	4.107032	4.0909	4.111177	4.111743	1.61319E-02	2.02770E-02	2.08430E-02
1.04	4.153582	4.1216	4.156089	4.156651	3.19819E-02	3.44892E-02	3.50513E-02
1.05	4.192997	4.1525	4.201055	4.201634	4.04973E-02	4.85554E-02	4.91347E-02
1.06	4.237143	4.1836	4.246097	4.246674	5.35431E-02	6.24971E-02	6.30736E-02
1.07	4.278533	4.2149	4.291212	4.291789	6.36334E-02	7.63121E-02	7.68890E-02
1.08	4.321807	4.2464	4.3364	4.336976	7.54070E-02	8.99997E-02	9.05752E-02
1.09	4.364085	4.2781	4.38166	4.382235	8.59852E-02	1.03560E-01	1.04135E-01
1.1	4.407133	4.3152	4.426993	4.427566	9.71327E-02	1.16993E-01	1.17566E-01
1.11	4.449872	4.3421	4.472397	4.472969	1.07769E-01	1.30297E-01	1.30869E-01
1.12	4.492957	4.3744	4.517871	4.518443	1.18550E-01	1.43471E-01	1.44042E-01
1.13	4.535988	4.4069	4.563416	4.563987	1.29080E-01	1.56517E-01	1.57087E-01
1.14	4.579194	4.4396	4.609032	4.609601	1.39594E-01	1.69432E-01	1.70001E-01
1.15	4.622451	4.4725	4.654716	4.655284	1.49951E-01	1.82217E-01	1.82785E-01
1.16	4.665834	4.5056	4.70047	4.701037	1.60234E-01	1.94870E-01	1.95437E-01
1.17	4.709295	4.5389	4.746293	4.746859	1.70395E-01	2.07393E-01	2.07959E-01
1.18	4.752861	4.5724	4.792183	4.792748	1.80461E-01	2.19784E-01	2.20349E-01
1.19	4.796515	4.6061	4.838142	4.838706	1.90415E-01	2.32042E-01	2.32606E-01
1.2	4.840267	4.64	4.884168	4.884731	2.00266E-01	2.44167E-01	2.44730E-01

Table 3: The table presents the result of schemes 3 and 4 for h = 0.01 compared with $y = -\frac{x^2}{4}$.

T	S3A	Y	S4A	S4B	ERR S3A	ERR S4A	ERR S4B
1	-0.25	-0.25	-0.25	-0.25	0	0	0
1.01	-0.25505	-0.25505	-0.25505	-0.25505	0	0	0
1.02	-0.26002	-0.2601	-0.26002	-0.26022	8.21000E-05	8.33000E-05	1.17900E-04
1.03	-0.26502	-0.26523	-0.26502	-0.26545	2.01200E-04	2.03800E-04	2.22300E-04
1.04	-0.27008	-0.2704	-0.27007	-0.27073	3.22000E-04	3.26000E-04	3.33400E-04
1.05	-0.27518	-0.27563	-0.27518	-0.27608	4.43400E-04	4.48800E-04	4.51000E-04
1.06	-0.28033	-0.2809	-0.28033	-0.28148	5.65400E-04	5.72200E-04	5.75300E-04
1.07	-0.28554	-0.28623	-0.28553	-0.28693	6.87800E-04	6.96200E-04	7.06200E-04
1.08	-0.29079	-0.2916	-0.29078	-0.29244	8.10800E-04	8.20600E-04	8.44000E-04
1.09	-0.29609	-0.29703	-0.29608	-0.29801	9.34200E-04	9.45500E-04	9.88600E-04
1.1	-0.30144	-0.3025	-0.30143	-0.30364	1.05790E-03	1.07090E-03	1.14020E-03
1.11	-0.30684	-0.30803	-0.30683	-0.30932	1.18200E-03	1.19660E-03	1.29880E-03
1.12	-0.31229	-0.3136	-0.31228	-0.31506	1.30650E-03	1.32270E-03	1.46470E-03
1.13	-0.31779	-0.31923	-0.31778	-0.32086	1.43120E-03	1.44900E-03	1.63780E-03
1.14	-0.32334	-0.3249	-0.32332	-0.32672	1.55620E-03	1.57570E-03	1.81820E-03
1.15	-0.32894	-0.33063	-0.32892	-0.33263	1.68150E-03	1.70260E-03	2.00620E-03
2.9	-2.10002	-2.1025	-2.09998	-2.3247	2.48050E-03	2.52410E-03	2.22197E-01
2.91	-2.11478	-2.11702	-2.11474	-2.34191	2.24690E-03	2.28480E-03	2.24885E-01
2.92	-2.12959	-2.1316	-2.12956	-2.35919	2.00980E-03	2.04200E-03	2.27593E-01
2.93	-2.14446	-2.14623	-2.14443	-2.37655	1.76860E-03	1.79480E-03	2.30321E-01
2.94	-2.15938	-2.1609	-2.15936	-2.39397	1.52320E-03	1.54350E-03	2.33069E-01
2.95	-2.17435	-2.17563	-2.17434	-2.41146	1.27410E-03	1.28820E-03	2.35838E-01
2.96	-2.18938	-2.1904	-2.18937	-2.42903	1.02110E-03	1.02900E-03	2.38626E-01
2.97	-2.20446	-2.20523	-2.20446	-2.44666	7.64100E-04	7.65600E-04	2.41435E-01
2.98	-2.2196	-2.2201	-2.2196	-2.46436	5.03300E-04	4.98000E-04	2.44263E-01
2.99	-2.23479	-2.23502	-2.2348	-2.48214	2.38400E-04	2.26500E-04	2.47112E-01

3.1 Graphical Representation of the Numerical Results

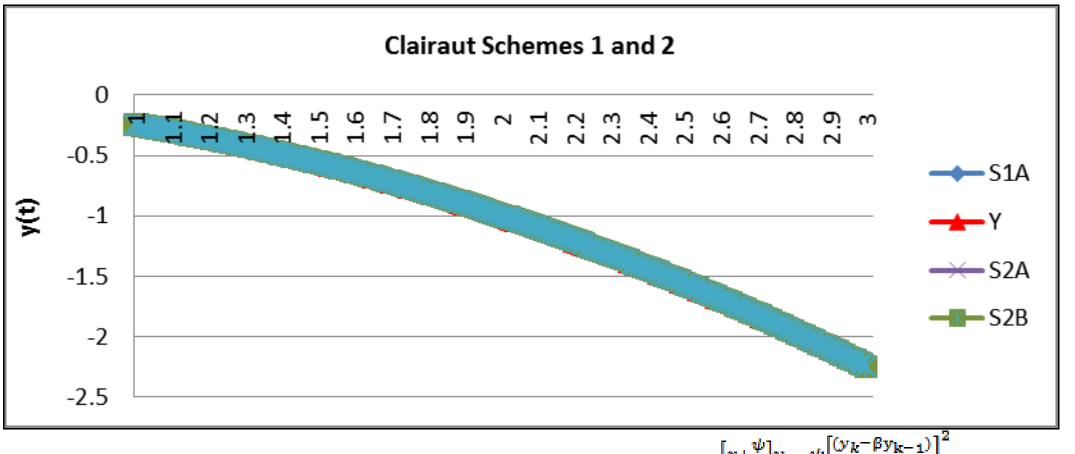

Figure 1: The graph presents the graph of Scheme $y_{k+1} = \frac{\left[x+\frac{\psi}{2}\right]y_k - \psi\left[\frac{(y_k-\beta y_{k-1})}{2\psi}\right]^2}{\left[x-\frac{\psi}{2}\right]}$

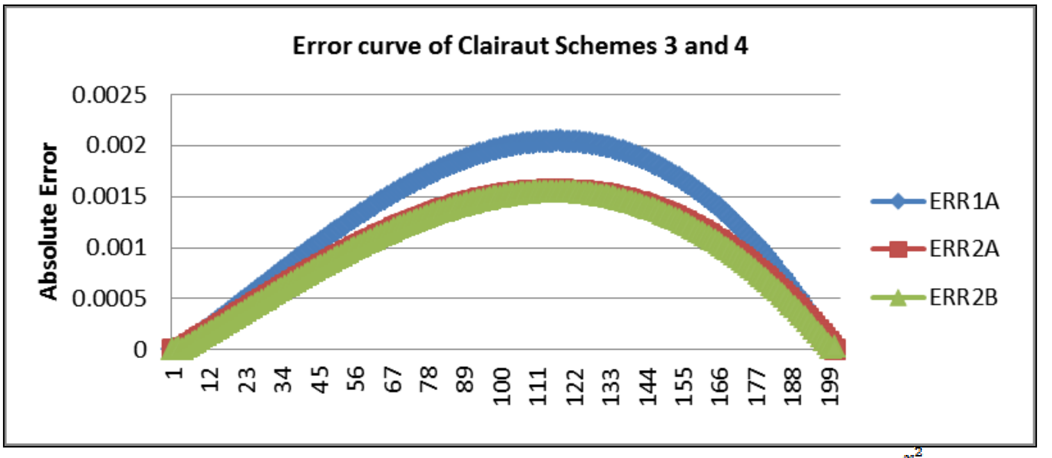

Figure 2: The graph presents the absolute Error of Deviation from $y = -\frac{x^2}{4}$

The solution curves for the schemes correspond to the curve of the singular solution and have the same monotonic behaviour. There is none of these three schemes that produce solution that thus not correspond to a solution of the original equation.

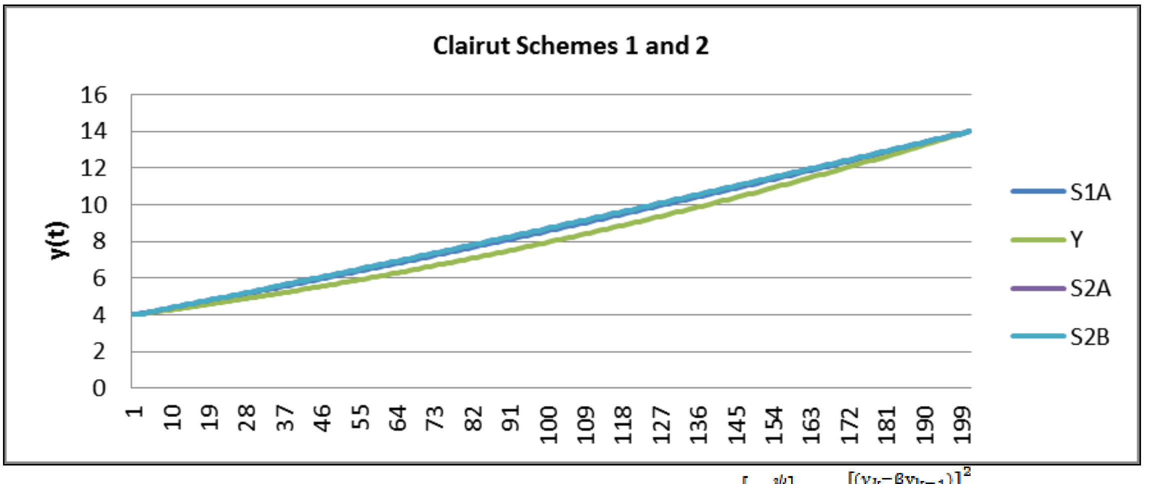

Figure 3: The graph presents the graph of Scheme $y_{k+1} = \frac{\left[x+\frac{\psi}{2}\right]y_k - \psi\left[\frac{(y_k-\beta y_{k-1})}{2\psi}\right]^2}{\left[x-\frac{\psi}{2}\right]}$.

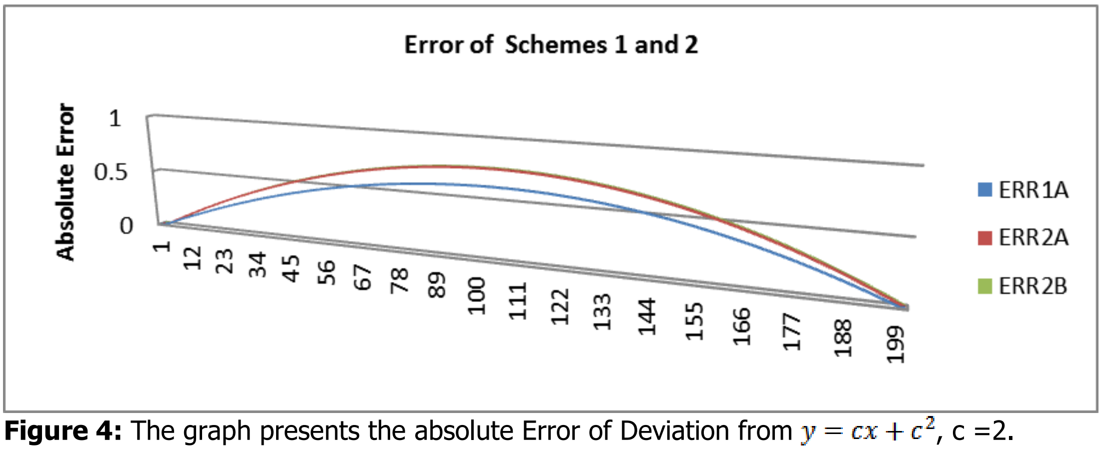

Figure 4: The graph presents the absolute Error of Deviation from $y = cx + c^2$, c =2.

The solution curves for the schemes correspond to the curve of the general solution and have the same monotonic behaviour. There is none of these three schemes that produce solution that thus not correspond to a solution of the original equation.

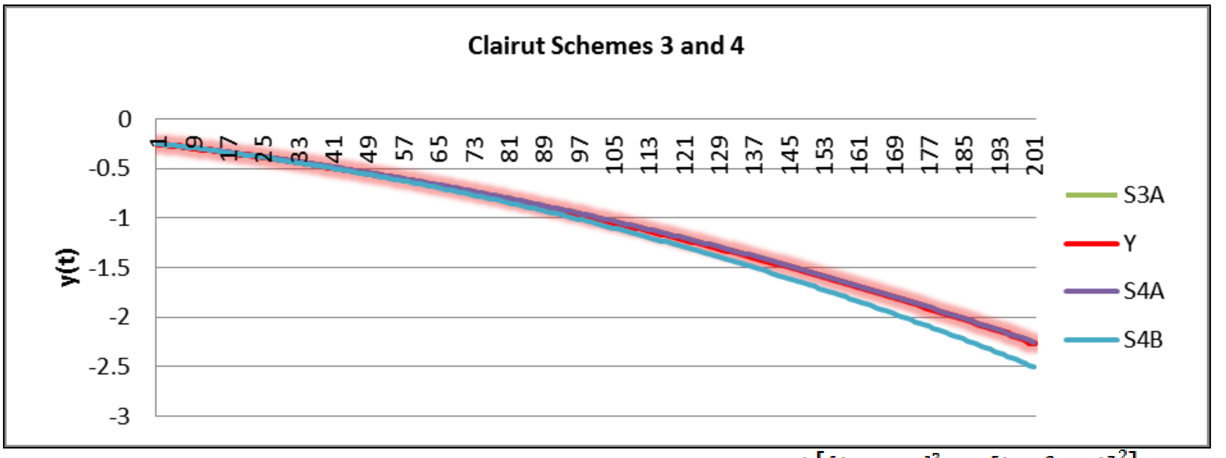

Figure 5: The graph presents the graph of Scheme $y_{k+1} = y_k + \frac{\psi}{x}\left[\frac{[(y_k+y_{k-1}]^2}{4\,(2y_k-y_{k-1})} - \left[\frac{(y_k-\beta y_{k-1})}{2\psi}\right]^2\right]$.

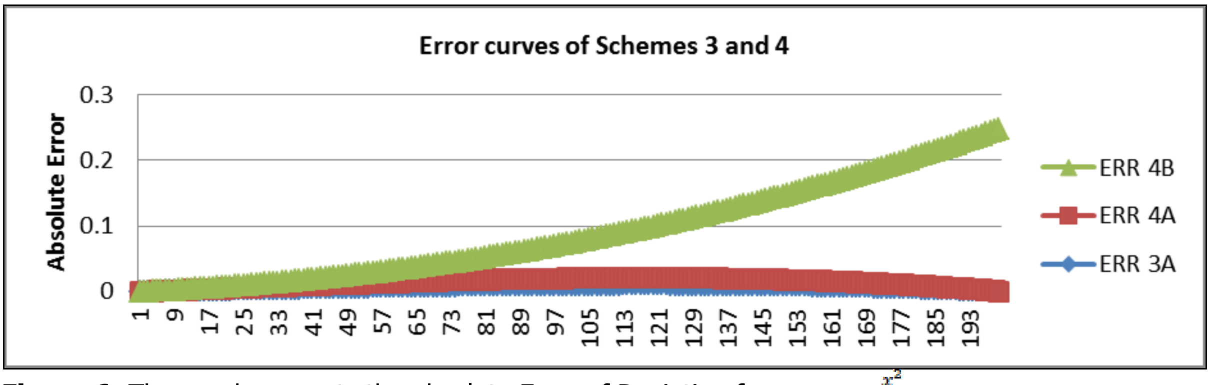

Figure 6: The graph presents the absolute Error of Deviation from $y = -\frac{x^2}{4}$

The solution curves for the schemes correspond to the curve of the singular solution and have the same monotonic behaviour. There is none of these three schemes produce solution that does not correspond to a solution of the original equation.

4. ERROR ANALYSIS

The absolute error of deviation for each scheme is depicted in Figures (2,4 and 6).
The Non-standard scheme proposed by Mickens (1994) [7] suggested the transformation of the Euler Method

$$y_{n+1} = y_n + hf(x_n y_n)$$

by replacing h with a function $\psi(h)$ which satisfies:

$$\psi(h) \rightarrow h + 0(h^2) as h \rightarrow 0$$

With a carefully selected function $\psi(h)$,the truncation error is of order(h^2) and the global truncation error is of order h.

The proponent of this method was much more interested in obtaining solution curves that behaves closely like the curve of the original equation. The Major achievement of this method is that, if the rules are satisfied then the schemes derived will possess some desirable qualitative properties like preservation of the hyperbolic and non-hyperbolic fixed points of the original equation.

5. CONCLUSION

The abysmal performance of schemes IB and 3B shows that the straight linear transformation of the non-linear ẏ component is grossly inadequate when used with denominator function $\psi = \sin(h)$.The introduction of β = cos(h) has acted as a factor that controls the behavior of the schemes as part of the renormalization of the discretization function as used in 2B and 4B (see Figure 1, Figure 3 and Figure 5). One of the major problems of Non-standard method is that there is no standard rule for choosing a particular denominator function. Though some denominator functions work well with certain class of equations, it cannot be generalized for a large class. This area is yet to be fully addressed but a lot of progress has been made. The solution curves for the schemes correspond to the curve of a particular solution of the Clairaut equation and the curves possess the same monotonic behaviour. There is none of these schemes that produce solution that does not correspond to a solution of the original equation. One of the advantages of Non-standard method is that if the rules are followed it creates schemes that are less susceptible to numerical instability. In particular it does not produce extraneous solution [7] and it produces solution that carry along the dynamics of the original equation as it is in the cases above. It is worthy to note that various other combinations of the transformations in Eq.(3-9) produce similar schemes with same qualitative behavior as the above. We can conclude that the schemes are suitable for numerical approximation of the original equation.

The major significance of this work is that this new class of schemes can be used to simulate the behavior of any dynamical system which can be represented by the Clairaut Equation.

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